



UNICUS
OLYMPIADS

#UnicusIsUnique

Sample Paper



Class 11

Unicus Mathematics Olympiad (UMO)

Time: 60 minutes

Pattern and Marking Scheme			
Section	Total Questions	Marks per Question	Total Marks
Classic Section	40	1	40
Scholar Section	10	2	20
Grand Total	50		60

Classic Section (Each Question is 1 Mark)

1. The letters of the word 'NATIONAL' are arranged at random. What is the probability that the last letter **will** be T?

a. $1/4$

b. $5/8$

c. $\frac{1}{8}$

d. $3/10$

2. 30 trees are planted in a straight line at intervals of 5 m. To water them, the gardener needs to bring water for each tree, separately from a well, which is 10 m from the first tree in line with the trees. How far will he have to walk in order to water all the trees beginning with the first tree? Assume that he starts from the well and he can carry enough water to water only one tree at a time.

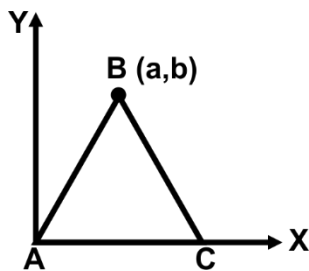
a. 4785 m

b. 4795 m

c. 4800 m

d. 4765 m

3. If the area of the given triangle is 20, what are the coordinates of point C?



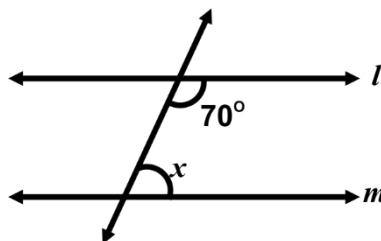
a. $(0, 40/a)$

b. $(a^2 + b^2, 0)$

c. $(20/b, 0)$

d. $(40/b, 0)$

4. Look at the figure given below. In the figure, if $l \parallel m$, then find the value of x :



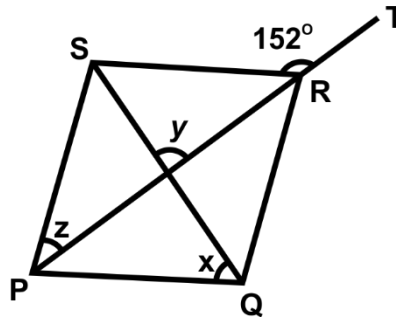
a. 120°

b. 110°

c. 90°

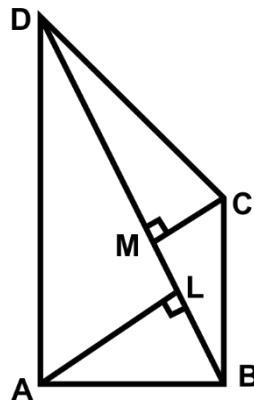
d. 98°

5. In the figure given below, PQRS is a rhombus in which the diagonal PR is produced to T. If $\angle SRT = 152^\circ$, find the value of x , y , and z :



- a. $x = 60^\circ$, $y = 90^\circ$, $z = 28^\circ$
 b. $x = 62^\circ$, $y = 80^\circ$, $z = 28^\circ$
 c. $x = 62^\circ$, $y = 90^\circ$, $z = 28^\circ$
 d. $x = 28^\circ$, $y = 90^\circ$, $z = 62^\circ$

6. In the adjoining figure, ABCD is a quadrilateral in which diagonal BD = 14 cm. If $AL \perp BD$ and $CM \perp BD$ such that $AL = 8$ cm and $CM = 6$ cm, then the area of quadrilateral ABCD is:



- a. 60 cm^2
 b. 72 cm^2
 c. 84 cm^2
 d. 98 cm^2

7. Molly purchased 23 bracelets for reselling at the rate of \$160 per bracelet. At what rate per bracelet should she sell the bracelets, so that profit earned is 15%?

- a. \$184
 b. \$186
 c. \$192
 d. \$198

8. Jack has some hens and some goats. If the total number of animal heads is 90 and the total number of animal feet is 248. What is the total number of goats Jack have?

- a. 32
 b. 36
 c. 34
 d. 56

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9. A two-digit number is three times the sum of its digits. If 45 is added to the number, its digits are interchanged. The sum of the digits of the number is:

a. 5
b. 7
c. 9
d. 11

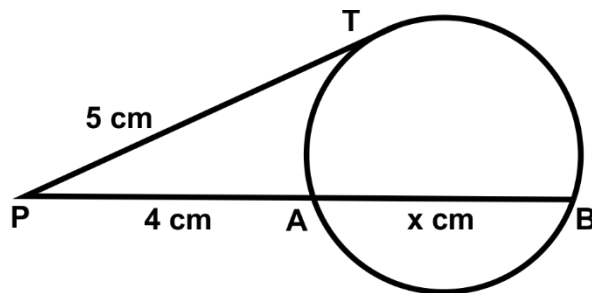
10. The solution of $2x + 3y = 2$ and $3x + 2y = 2$ can be represented by a point in the coordinate plane in:

a. 1st quadrant
b. 2nd quadrant
c. 3rd quadrant
d. 4th quadrant

11. If the 10th term of the sequence $a, a - b, a - 2b, a - 3b, \dots$ is 20 and its 20th term is 10, then its x^{th} term is:

a. $10 - x$
b. $20 - x$
c. $29 - x$
d. $30 - x$

12. In the given figure, PAB is secant and PT is a tangent to the circle from P. If $PT = 5$ cm, $PA = 4$ cm and $AB = x$ cm, then what will be the value of x ?



a. 2.5 cm
b. 2.6 cm
c. 2.25 cm
d. 2.75 cm

13. Which of the following is a rational number?

a. Sum of $2 + \sqrt{3}$ and its inverse
b. Square root of 18
c. Square root of $7 + 4\sqrt{3}$
d. Square root of 9

14. If α, β, γ are the zeroes of the polynomial $f(x) = x^3 - 5x^2 - 2x + 24$ such that $\alpha\beta = 12$, then which of the following options is/are true?

a. $\alpha + \beta = 7$
b. $\alpha - \beta = +1$
c. $\gamma = -2$
d. All of these

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- 15.** Peter started cycling along the boundaries of a square field ABCD from corner point A. After half an hour, he reached the corner point C, diagonally opposite to A. If his speed was 8 km/hr, then the area of the field is:

- a. 64 km^2 b. 8 km^2
c. 4 km^2 d. 16 km^2

- 16.** What is the ratio of the areas of circles inscribed and circumscribed in a square?

- a. 1: $\sqrt{2}$
c. 1: 2

17. The perimeters of a square and a rectangle are equal. If their areas are respectively $A \text{ m}^2$ and $B \text{ m}^2$, then which of the following is a true statement?

- a. $A < B$
c. $A > B$
- b. $A < B$
d. None of these

- 18.** How many odd numbered pages are there in a book of 1089 pages?

- [illegible]

- 19.** A man has to pay \$975 in yearly installment, each installment is less than the earlier one by \$5. The amount of the first installment is \$100. At what time, the entire amount will be paid?

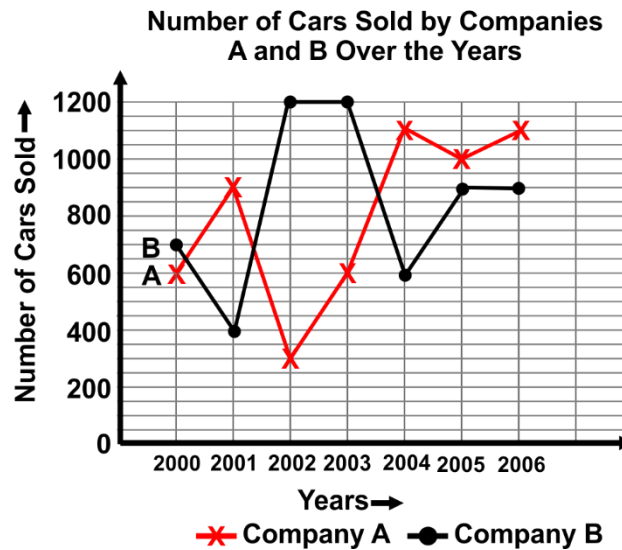
- a. 10 years b. 12 years
c. 15 years d. 25 years

- 20.** Study the following table carefully and answer the question given below:
What is the average number of failed students from class VII for the given years?

Number of Students Passed and Failed in 5 Classes of a School Over the Years										
Classes / Years	VI		VII		VIII		IX		X	
	Pass	Fall	Pass	Fall	Pass	Fall	Pass	Fall	Pass	Fall
2001	50	42	76	14	58	18	65	17	48	23
2002	60	19	95	22	71	30	75	12	76	28
2003	45	13	61	19	49	15	48	08	74	20
2004	58	21	75	25	80	28	60	11	84	14
2005	55	18	66	29	59	26	70	13	65	17
2006	68	31	54	38	77	34	82	21	55	14
Total	336	144	427	147	394	151	400	82	402	116

- a. 27.5 b. 28
c. 26.5 d. 24.5

21. Which of the following combinations of year and percentage rise in sales from the previous year for Company A is correct? (percentage rounded off to two decimal places)



- | | |
|------------------|------------------|
| a. 2003 - 99.85% | b. 2004 - 83.33% |
| c. 2001 - 43.21% | d. 2006 - 7.68% |

22. Express $[x^2 - x - 6/3x^2 + 7x + 2] \div [3x^2 - 9x/3x^2 + 7x + 2]$ as rational expression in its lowest terms:

- | | |
|-----------------|-----------------|
| a. $(x - 2)/3x$ | b. $(x + 2)/2x$ |
| c. $(x + 2)/3x$ | d. $(x - 2)/2x$ |

23. If the L.C.M. of $(x^2 + 3x)$ $(x^2 + 3x + 2)$ and $(x^2 + kx + 8)$ $(x^2 + 5x + 6)$ is $x(x + 1)(x + 2)^2(x + 3)(x + 4)$, then the value of K is:

- | | |
|------|------|
| a. 5 | b. 6 |
| c. 8 | d. 7 |

24. If $A = (x + 1)/(2x + 1)$, $B = (2x - 1)/(x + 2)$, and $C = (4x - 7)/(2x^2 + 5x + 2)$, then find the value of $(4A - B - C)$:

- | | |
|-----------------------|-----------------------|
| a. $(x - 2)/(2x + 1)$ | b. $8/(2x - 1)$ |
| c. $8/(2x + 1)$ | d. $(2x - 1)/(x + 2)$ |

25. Sam spends 23% of the amount of money on an insurance policy, 33% on food, 19% on children's education and 16% on recreation. He deposits the remaining amount of \$504 in the bank. How much total amount does he spend on food and insurance policy together?

- | | |
|-----------|-----------|
| a. \$3200 | b. \$3126 |
| c. \$3136 | d. \$3048 |

26. Sara purchased an item for \$9600 and sold it for a loss of 5%. From that money, she purchased another item and sold it for a gain of 5%. What is her overall gain/loss?

- | | |
|-----------------|-------------------|
| a. Loss of \$36 | b. Profit of \$24 |
| c. Loss of \$24 | d. Profit of \$36 |
-

27. If an electricity bill is paid before the due date, one gets a reduction of 4% on the amount of the bill. By paying the bill before the due date, a person got a reduction of \$13. The amount of his electricity bill was:

- | | |
|----------|----------|
| a. \$125 | b. \$225 |
| c. \$325 | d. \$425 |
-

28. A student walks from his house at $2\frac{1}{2}$ km/h and reaches his school late by 6 min. The next day, he increases his speed by 1 km/h and reaches 6 min before school time. How far is the school from his house?

- | | |
|---------------------|----------------------|
| a. $\frac{5}{4}$ km | b. $\frac{7}{4}$ km |
| c. $\frac{9}{4}$ km | d. $\frac{11}{4}$ km |
-

29. Two persons A and B start simultaneously from two places c km apart and walk in the same direction. If A travels at the rate of p km/h and B travels at the rate of q km/h, then A has travelled before he overtakes B a distance of:

- | | |
|--------------------|--------------------|
| a. $qc/(p + q)$ km | b. $pc/(p - q)$ km |
| c. $qc/(p - c)$ km | d. $pc/(p + q)$ km |
-

30. Two trains are running 40 km/h and 20 km/h respectively, in the same direction. The fast train completely passes a man sitting in the slow train in 5 s. The length of the fast train is:

- | | |
|----------------------|---------|
| a. $23\frac{2}{9}$ m | b. 27 m |
| c. $27\frac{7}{9}$ m | d. 23 m |
-

31. Find the equation of the line which makes equal intercepts on the axis and passes through the point (4, 5):

- | | |
|------------------|-----------------|
| a. $x + y = 9$ | b. $x + 2y = 7$ |
| c. $2x + 2y = 7$ | d. $2x + y = 7$ |
-

32. The angles of depression of two ships from the top of a light-house are 60° and 45° towards East. If the ships are 300 m apart, the height of the light-house is:

- | | |
|--------------------------|--------------------------|
| a. $200(3 + \sqrt{3})$ m | b. $250(3 + \sqrt{3})$ m |
| c. $150(3 + \sqrt{3})$ m | d. $160(3 + \sqrt{3})$ m |
-

33. The mean of 20 observations is 17. On checking it was found that the two observations were wrongly copied as 3 and 6. If wrong observations are replaced by correct values 8 and 9, then what is the correct mean?

- | | |
|---------|---------|
| a. 17.4 | b. 16.6 |
| c. 15.8 | d. 14.2 |
-

34. If α is in the first quadrant such that $\tan^2 \alpha = 8/7$, then the value of $[(1 + \sin \alpha)(1 - \sin \alpha)] / [(1 + \cos \alpha)(1 - \cos \alpha)]$ is:

- | | |
|----------|------------|
| a. $7/8$ | b. $8/7$ |
| c. $7/4$ | d. $64/49$ |
-

35. The angles of elevation of the top of a tower from two points A and B lying on the horizontal through the foot of the tower are respectively 15° and 30° . If A and B are on the same side of the tower and $AB = 48$ m, then the height of the tower is: ($\tan 15^\circ = 2 - \sqrt{3}$)

- | | |
|-------------------|---------|
| a. $24\sqrt{3}$ m | b. 24 m |
| c. $28\sqrt{3}$ m | d. 96 m |
-

36. A conical flask has been radius a cm and height h cm. It was completely filled with milk. The milk is poured into a cylindrical thermos flask whose base radius is p cm. What will be the height of the solution level in the flask?

- | | |
|-------------------|-------------------|
| a. $a^2h/3p^2$ cm | b. $3hp^2/a^2$ cm |
| c. $p^2h/3h^2$ cm | d. $3a^2/hp^2$ cm |
-

37. If the sum of the roots of a quadratic equation is 3 and the product is 2, then the equation is:

- | | |
|-----------------------|-----------------------|
| a. $2x^2 - x + 3 = 0$ | b. $x^2 - 3x + 2 = 0$ |
| c. $x^2 + 3x + 2 = 0$ | d. $x^2 - 3x - 2 = 0$ |
-

38. The sum of all interior angles of a regular polygon is twice the sum of all its exterior angles. The polygon is:

- | | |
|---------------|--------------|
| a. an octagon | b. a nonagon |
| c. a decagon | d. a hexagon |
-

39. Two trains start from A and B at the same time and move towards B and A, respectively. They take 8 h and 6 h for their journey, respectively. Two hours after they start, the train from B meets with an accident and thereafter moves at half its normal speed. How many hours after starting will the two trains meet?

- | | |
|-----------------|-----------------|
| a. $24/7$ hours | b. $25/4$ hours |
| c. 4 hours | d. 6 hours |
-

- 40.** In a bag, there are 6 red balls and 9 green balls. Two balls are drawn at random. What is the probability that at least one of the balls drawn is red?
- a. $\frac{29}{35}$ b. $\frac{7}{15}$
- c. $\frac{23}{35}$ d. $\frac{17}{35}$

Scholar Section (Each Question is 2 Marks)

- 41.** A toy factory manufactured a batch of electronic toys. If the toys were packed in boxes of 115 each, 13 boxes would not be filled completely. If the toys were packed in boxes of 65 each, 22 such boxes would not be enough to pack all of them. Coincidentally, in the end, the toys were packed in n boxes containing n toys each, without any remainder. The total number of toys was:
- a. 1424 b. 1434
c. 1444 d. 1454

- 42.** Two persons P and Q lent certain amounts at the same rate of interest for 2 years and 3 years, respectively, under compound interest. Their final amounts are in the ratio 3: 5. Q's amount at the end of the first year was \$8500 and P earned an interest of \$510 for the first year. Find the ratio of their principals:
- a. 33: 50 b. 27: 47
c. 35: 61 d. 22: 51

- 43.** A man arranges to pay off a debt of \$3600 in 40 annual installments which are in A.P. When 30 of the installments are paid, he dies leaving one-third of the debt unpaid. The value of the 8th installment is:
- a. \$35 b. \$61
c. \$51 d. \$99

- 44.** A pencil box has 25 pencils and 5 erasers. Three things are drawn out of the pencil box. Find the probability that the three things drawn are either only pencils or only erasers.
- a. $\frac{{}^{25}\text{C}_3}{{}^{30}\text{C}_3}$
- b. $(\frac{{}^{25}\text{C}_3 + {}^5\text{C}_3}{{}^{30}\text{C}_3})$
- c. $\frac{{}^5\text{C}_3}{{}^{30}\text{C}_3}$
- d. $\frac{{}^{25}\text{C}_3 \times {}^5\text{C}_3}{{}^{30}\text{C}_3}$

- 45.** Which of the following is correct in respect of the number 1729?
- a. It cannot be written as the sum of the cubes of two positive integers.
 - b. It can be written as the sum of the cubes of two positive integers in one way only.
 - c. It can be written as the sum of the cubes of two positive integers in two ways only.
 - d. It can be written as the sum of the cubes of two positive integers in three ways only.

46. The base AB of two equilateral triangles ABC and ABC' with $2a$ lies along the x-axis such that the mid-points of AB is the origin. Find the ordinates of the vertices C and C' of the triangles:

- a. $(\sqrt{3}a, 0)$ & $(-\sqrt{3}a, 0)$ b. $(0, \sqrt{4}a)$ & $(0, -\sqrt{4}a)$
 c. $(0, \sqrt{3}a)$ & $(0, -\sqrt{3}a)$ d. $(\sqrt{4}a, 0)$ & $(-\sqrt{4}a, 0)$
-

47. In solving a problem that reduces to a quadratic equation, one student makes a mistake in the constant term and obtains 8 and 2 for roots. Another student makes a mistake only in the coefficient of the first-degree term and finds -9 and -1 for roots. The correct equation is:

- a. $x^2 - 10x + 9 = 0$ b. $x^2 + 10x + 9 = 0$
 c. $x^2 - 10x + 16 = 0$ d. $x^2 - 8x - 9 = 0$
-

48. Consider the following statements:

1. If 3θ is an acute angle such that $\sin 3\theta = \cos 2\theta$, then the measurement of θ in radian equals to $\pi/10$.
2. One radian is the angle subtended at the centre of a circle by an arc of the same circle whose length is equal to the diameter of that circle.

Which of the above statement(s) is/are correct?

- a. Only I b. Only II
 c. Both I and II d. Neither I nor II
-

49. One fourth of a herd of cows is in the forest. Twice the square root of the herd has gone to mountains and on the remaining 15 are on the banks of a river. The total number of cows is:

- a. 6 b. 100
 c. 63 d. 36
-

50. Match the statements of Column A with the values of Column B:

Column A		Column B	
1.	The maximum value of $-3x^2 + 12x + 8$ is attained at x is	a.	6
2.	One of the roots quadratic equation, $x^2 - 6x + 9 = 0$ is	b.	5
3.	If $5x + ky = 22$ and $kx + 5y = 23$ are inconsistent, then k can be	c.	3
4.	The sum of the roots of $\frac{5}{3}x^2 - 10x + 2 = 0$ is	d.	2

- a. 1 - (a), 2 - (b), 3 - (d), 4 - (c) b. 1 - (d), 2 - (c), 3 - (b), 4 - (a)
 c. 1 - (a), 2 - (d), 3 - (c), 4 - (b) d. 1 - (c), 2 - (b), 3 - (d), 4 - (a)
-

Answer Key

1.	c	2.	b	3.	d	4.	b	5.	c	6.	d	7.	a
8.	c	9.	c	10.	a	11.	d	12.	c	13.	a	14.	d
15.	c	16.	c	17.	c	18.	c	19.	c	20.	d	21.	b
22.	c	23.	b	24.	c	25.	c	26.	c	27.	c	28.	b
29.	b	30.	c	31.	a	32.	c	33.	a	34.	a	35.	b
36.	a	37.	b	38.	d	39.	c	40.	c	41.	c	42.	a
43.	c	44.	b	45.	c	46.	c	47.	a	48.	a	49.	d
50.	b												